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ARMY ELECTRONICS COMMAND  
FORT MONMOUTH, NEW JERSEY

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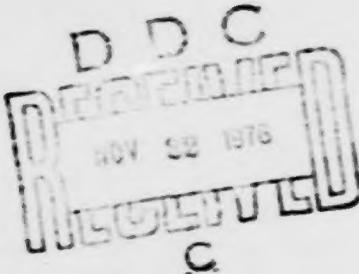
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RESEARCH AND DEVELOPMENT TECHNICAL REPORT  
ECOM-5596

THE DEPTH OF THE SURFACE  
BOUNDARY LAYER



By

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June 1976

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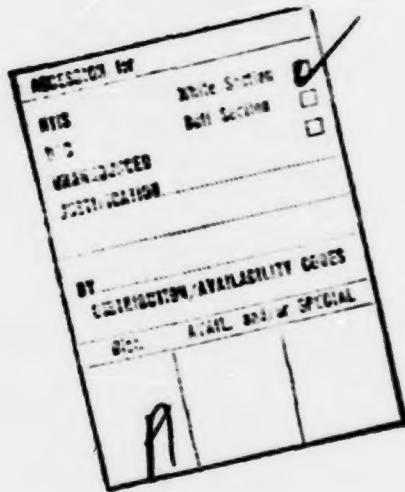
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## SUMMARY

In general, the approach presented to determine the depth of the surface boundary layer is simple and relatively straightforward. The depths of the surface boundary layer as indicated in Figure 1 are representative averages, while the depths shown in the remaining figures are relative owing to the smoothing techniques utilized. The binomial filtering method has a tendency to reduce peaks drastically. The results obtained appear to be reasonable and agree with the experimental determinations of Lumley and Panofsky.



## CONTENTS

	<u>Page</u>
SUMMARY	1
INTRODUCTION	3
DYNAMIC SIMILARITY IN THE SURFACE BOUNDARY LAYER	3
THE DEPTH OF THE SURFACE BOUNDARY LAYER	5
DISCUSSION	7
REFERENCES	8

## INTRODUCTION

Within the planetary boundary layer known as the friction zone of the atmosphere, a sublayer exists where the heat and momentum fluxes are considered to be invariant with height. This surface boundary layer is further characterized by a scalar Reynolds stress along the mean wind, the direction of which is also invariant with height since coriolis effects may be assumed to be insignificant. The depth of the surface boundary layer can vary from a few meters to perhaps a hundred or more meters, depending upon surface roughness, stability, and windspeed. In turn, these parameters and the thickness of the surface boundary layer control the maximum values of the eddy conductivity and eddy viscosity that occur in the planetary boundary layer.

A method for determining the thickness (or depth) of the surface boundary layer can be easily developed from the dynamic similarity theory of Obukhov [1], in terms of measurable and calculated micrometeorological parameters. The scheme is simple, readily implemented, and allows rapid determination of the depth of the surface boundary layer and exchange coefficient maxims for the planetary boundary layer.

### DYNAMIC SIMILARITY IN THE SURFACE BOUNDARY LAYER

Obukhov [1] postulated that heat and momentum fluxes in the surface boundary layer were dynamically similar. Drawing on Richardson's [2] criterion and the Schmidt [3] exchange coefficient hypothesis, Obukhov reasoned that heat and momentum transfer processes in the surface boundary layer could be stated as

$$H = \frac{K_H}{K_M} \theta (Ri) c_p \rho \ell^2 \left( \frac{\partial \bar{V}}{\partial z} \right) \left( \frac{\partial \bar{\theta}}{\partial z} \right) \quad (1)$$

and

$$\tau = -\theta (Ri) \rho \ell^2 \left( \frac{\partial \bar{V}}{\partial z} \right)^2 \quad (2)$$

where  $H$  is the vertical heat flux,  $c_p$  the specific heat of air at constant pressure,  $\bar{\theta}$ , the potential temperature,  $\bar{V}$  the mean windspeed,  $K_H/K_M$  the ratio of the exchange coefficients for heat and momentum,  $\rho$  density,  $\tau$  the Reynolds stress,  $\theta (Ri)$  a monotonously decreasing universal function,  $\ell$  the Prandtl [4] mixing length, and  $z$  height. Paralleling, and in conjunction with Equations (1) and (2), the conjugate laws for the wind and temperature profiles in differential form may be written as:

$$\frac{\partial \bar{V}}{\partial z} = \frac{u_*}{kz} \theta_M \quad (3)$$

$$K_M = k u_* z \theta_M^{-1} \quad (4)$$

$$\frac{\partial \theta}{\partial z} = \frac{T^*}{z} \theta_H \quad (5)$$

$$K_H = k u_* z \theta_H^{-1} \quad (6)$$

where  $u_*$  is a friction velocity,  $k$  Karman's constant,  $\theta_M = [\theta(Ri)]^{-\frac{1}{2}}$ ,  $\theta_H = [\theta(Ri)]^{-1}$ , and  $T^*$  a scaling temperature defined as

$$T^* = - \frac{1}{k u_*} \frac{H}{c_p \rho} \quad (7)$$

The gradient Richardson number,  $Ri$ , is generally written as

$$Ri = \frac{g}{\theta} \frac{\partial \theta / \partial z}{(\partial \bar{V} / \partial z)^2} \quad (8)$$

where  $g$  is the gravitational acceleration. Operating upon Equation (8) with respect to  $\theta(Ri)$ , Obukhov found that Equation (8) could be stated as

$$Ri = \frac{K_M}{K_H} \frac{kgzH}{u_*^3 c_p \rho \theta} [\theta(Ri)]^{\frac{1}{2}}. \quad (9)$$

If Equation (9) is differentiated with respect to  $z$ , what is now known as the Obukhov scaling length is represented by

$$L = - \frac{u_*^3 c_p \rho \theta}{kgH} \quad (10)$$

where  $L$  is a characteristic length representative of and proportional to the depth of the surface boundary layer. By definition it may also be stated that

$$\frac{z}{L} = \theta_M Ri \frac{K_H}{K_M} \quad (11)$$

which may also be stated as  $z/L = \theta_M R_f$  where  $R_f$  is the flux form of the Richardson number.

A universal form of the wind profile may now be developed from Equations (3) and (11). Writing Equation (3) as

$$\frac{\partial \bar{V}}{\partial z} = \frac{u_*}{kz} \frac{z/L}{R_f}, \quad (12)$$

Then adding and subtracting 1 to  $z/L R_f^{-1}$  and multiplying and dividing by  $z/L$ ,

$$\frac{\partial \bar{V}}{\partial z} = \frac{u_*}{kz} \left[ 1 + \frac{z}{L} \frac{z/L - R_f}{R_f z/L} \right]. \quad (13)$$

Defining  $z/L - R_f (R_f z/L)^{-1}$  as an arbitrary variable  $\beta$ , then integrating Equation (13),

$$\Gamma = \frac{u_*}{k} \left[ \ln \frac{z}{z_0} + \bar{\beta} \frac{z}{L} \right] \quad (14)$$

where  $\bar{\beta}$  is the average  $\beta$  over the layer  $z_g = (z_1 z_2)^{1/2}$  and  $z_0$  is the roughness length. Since  $R_f = R_i K_H/K_M$ , then

$$1 + \frac{z}{L} \frac{z/L - R_f}{R_i z/L} \frac{K_M}{K_H} = 1 + \bar{\beta} \frac{z}{L} = \frac{K_H}{K_M} (\beta_M - 1)$$

and

$$\bar{V} = \frac{u_*}{k} \left[ \ln \frac{z}{z_0} + \frac{K_H}{K_M} (\beta_M - 1) \right] \quad (15)$$

which is valid for both stable and unstable thermally stratified flow in the surface boundary layer.

#### THE DEPTH OF THE SURFACE BOUNDARY LAYER

Lumley and Panofsky [5] suggest that the depth of the surface boundary layer may be empirically determined from

$$h = 20 \tau_0 \quad (16)$$

where  $h$  is in meters and  $\tau_0$  is the surface stress. It may be assumed that  $\tau_0 = \tau = \rho u_*^2$ , the Reynolds stress. Equation (16) is based upon the assumption that  $\tau$  varies about 20 percent from  $z_0$  to  $h$ .

A more precise argument can be developed from Equation (3). If Equation (3) is written in finite difference form for the unstable regime as

$$\frac{\Delta V}{h \Delta \ln z} = \frac{u_*}{k} \frac{\theta_M}{Ri} L^{-1} \quad (17)$$

where  $z = h = L \cdot Ri$ , then

$$h = L \frac{k}{u_*} \frac{Ri}{\theta_M} \frac{\Delta V}{\Delta \ln z} \quad (18)$$

If  $\Delta V$  the wind gradient at the geometric mean height  $h$  is assumed to occur over the layer  $\Delta \ln z = \ln e = 1$ , then  $\Delta V^{-1} = Ri \theta_M^{-1}$  and

$$h = mL \quad (19)$$

where  $m = \left| k u_*^{-1} \right|$  the slope of the normalized diabatic profile.

In thermally stratified stable flow, Equation (3) is written as

$$h = L \frac{k}{u_*} Ri \frac{\Delta V}{\Delta \ln z} \quad (20)$$

owing to the assumption that all fluxes are a result of mechanical turbulence only. This leads to  $K_H = K_M$  and  $z/L = Ri \theta_M$ . If  $\Delta \ln z$  is again unity, then  $Ri \Delta V = \beta^{-1}$  and

$$h = mL \beta^{-1} = mL \frac{\theta_M}{15} \quad (21)$$

since for stable flow, Hansen [6] has shown that

$$\frac{z}{L} = Ri + 15 Ri^2, \quad (22)$$

$$\theta_M = 1 + 15 Ri, \quad (23)$$

and

$$\beta = 15 \theta_M^{-1}. \quad (24)$$

If  $h/L = \text{Ri } \theta_M$ , then from Equation (21)

$$\text{Ri}(h) = \frac{m}{15} \quad (25)$$

and

$$h = L \left[ \text{Ri}(h) + 15 \text{Ri}^2(h) \right] \quad (26)$$

where the subscript denotes the height of the parameter evaluation.

#### DISCUSSION

Equations (19) and (21), the primary formulae for calculating the depth of the surface boundary layer, were evaluated by using experimental data extracted from studies reported on by Lettau and Davidson [7], Barad [8], Swinbank [9] and Izumi [10]. Generally, these data were observed in relatively stationary conditions and mostly over terrain that possessed a high degree of homogeneity. Of the 493 profiles available, only 36 were unusable because of the extremely low windspeeds reported.

Gradient Richardson numbers, scaling lengths, profile slopes, Reynolds stresses, and the dimensionless shears were calculated for each profile. Surface boundary layer depths were determined for both the stable and unstable flow regimes. These data were then averaged as a geometric progression as a function of  $L$  and are shown in Figure 1. The curve representing  $h$ , calculated by using Equation (16), is also given. The agreement between Equation (16) and  $h$  as determined from Equations (19) and (21) is considered to be good. Included for definitive purposes are the smoothed average windspeeds observed over the entire stability range. The divergence of the two curves over the range  $-55 < L < 222$  meters is attributed to the fact that the profiles observed in this stability range were observed near sunrise and sunset under light wind conditions and indifferent thermal stratification. The dashed portions of the curves through neutral conditions are suggested shapes owing to a lack of data near  $L = \infty$ .

Surface boundary layer depth with respect to windspeed at 2 m above the surface is shown in Figure 2. Both  $V$  and  $h$  were smoothed with a five-point binomial filter over 24 hours to obtain this representation. It is apparent that surface boundary layer depth is somewhat of a function of windspeed and, in turn, the Reynolds stresses.

Figures 3a and 3b present the surface boundary layer depth as a function of  $L$  and  $L^{-1}$ , respectively. Again, these data were smoothed by using the binomial filter over 24 hours. Figures 3a and 3b show that in near neutral conditions, the depth of the surface boundary layer approaches the depth of the planetary boundary layer. If this is so, then the limit of the Obukhov [1] length will not be infinity but the overall depth of the planetary boundary layer in adiabatic flow conditions.

## REFERENCES

1. Obukhov, A. M., 1946, "Turbulence in an Atmosphere of Nonhomogeneous Temperature," Trans. Inst. Theor. Geophys., USSR 1, pp 95-115.
2. Richardson, L. F., 1920, "The Supply of Energy from and to Atmospheric Eddies," Proc. Roy. Soc. A97, pp 354-373.
3. Schmidt, W., 1925, Der Massenaustausch in Freier Luft und Verwandte Erscheinungen, Probleme der kosmischen Physik, Hanburg, Verlag Von Henpi Grand.
4. Prandtl, L., 1934, "The Mechanics of Viscous Fluids," Aerodynamic Theory, III, G. Division and W. F. Durand (Ed), Berlin.
5. Lumley, J. L. and H. A. Panofsky, 1964, The Structure of Atmospheric Turbulence, J. Wiley and Sons, New York, 239 pp.
6. Hansen, F. V., 1976, "The Critical Richardson Number," to be published.
7. Lettau, H. H. and B. Davidson, 1957, Exploring the Atmosphere's First Mile, I and II, Pergamon Press, New York.
8. Barad, M. L., 1958, "Project Prairie Grass, A Field Program in Diffusion," Geophysical Research Paper No. 59, Air Force Cambridge Research Center, Bedford, Massachusetts, II, 209 pp.
9. Swinbank, W. C., 1964, "The Exponential Wind Profile," Quart. J. Roy. Meteorol Soc. 90, pp 119-135.
10. Izumi, Y., 1971, "Kansas 1968 Field Program Data Report," Environmental Research Paper No. 379, Air Force Cambridge Research Laboratories, Bedford, Massachusetts, 79 pp.

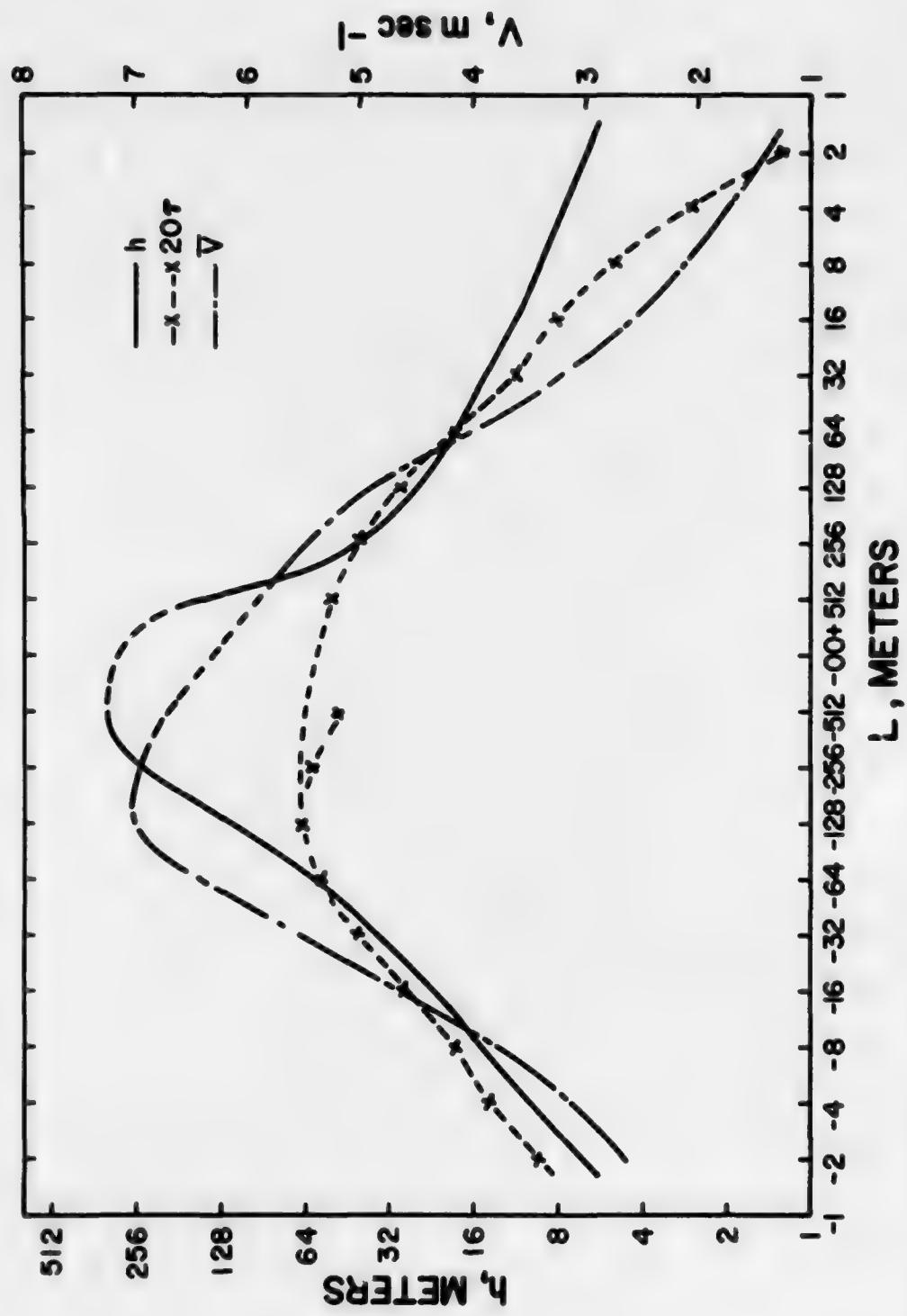


Figure 1. Surface boundary layer depth as a function of the scaling length  $L$ .

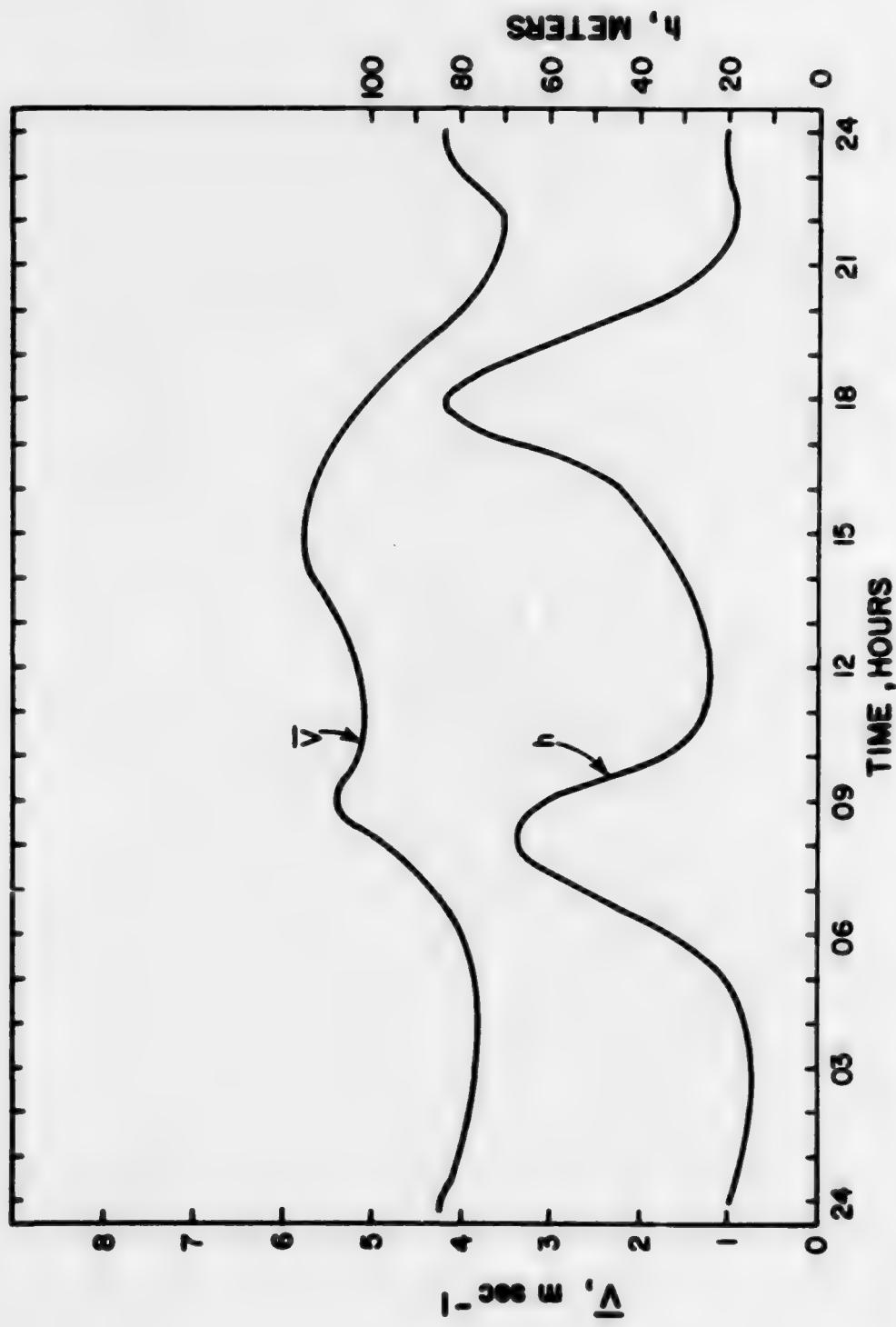


Figure 2. Mean windspeed and surface boundary layer depth as a function of time.

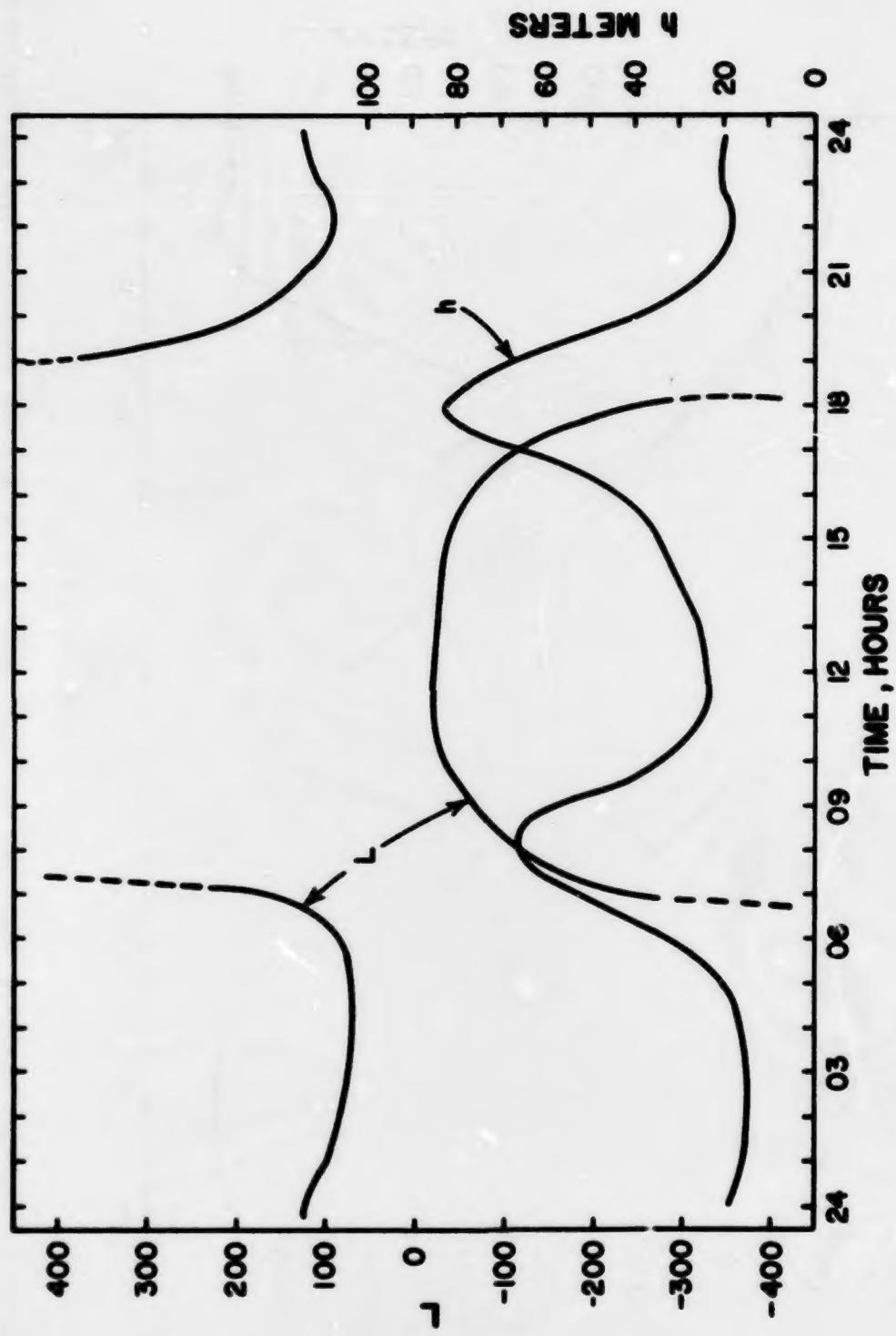


Figure 3a. Surface boundary layer depth as a function of time and the scaling length  $L$ .

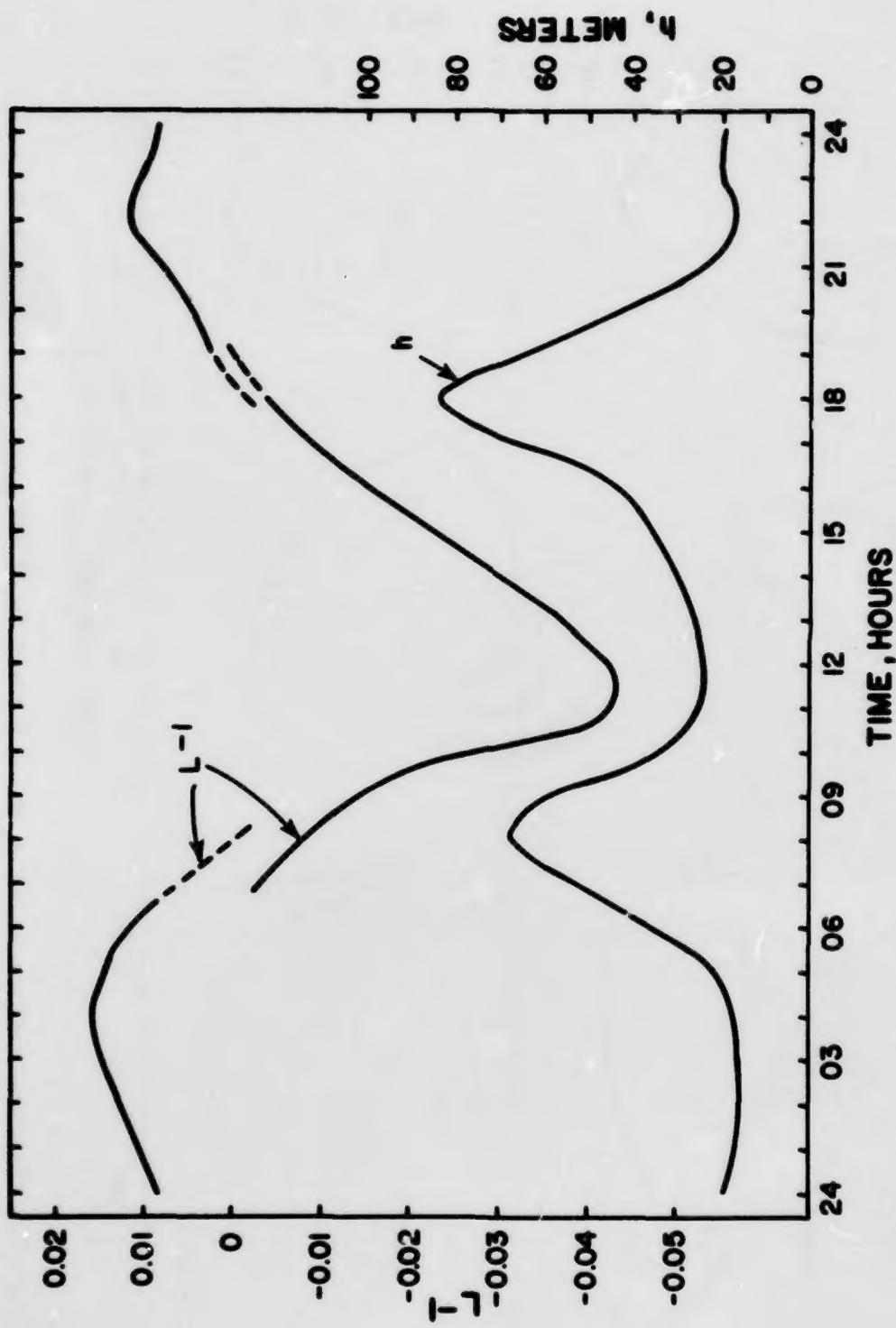


Figure 3b. Surface boundary layer depth as a function of time and the inverse of the scaling length.

## ATMOSPHERIC SCIENCES RESEARCH PAPERS

1. Lindberg, J.D., "An Improvement to a Method for Measuring the Absorption Coefficient of Atmospheric Dust and other Strongly Absorbing Powders," ECOM-5565, July 1975.
2. Avara, Elton, P., "Mesoscale Wind Shears Derived from Thermal Winds," ECOM-5566, July 1975.
3. Gomez, Richard B. and Joseph H. Pierluissi, "Incomplete Gamma Function Approximation for King's Strong-Line Transmittance Model," ECOM-5567, July 1975.
4. Bifano, A.J. and B.F. Engebos, "Ballistic Wind Weighting Functions for Tank Projectiles," ECOM-5568, August 1975.
5. Taylor, Fredrick J., Jack Smith, and Thomas H. Pries, "Crosswind Measurements through Pattern Recognition Techniques," ECOM-5569, July 1975.
6. Walters, D.L., "Crosswind Weighting Functions for Direct-Fire Projectiles," ECOM-5570, August 1975.
7. Duncan, Louis D., "An Improved Algorithm for the Iterated Minimal Information Solution for Remote Sounding of Temperature," ECOM-5571, August 1975.
8. Robbiani, Raymond L., "Tactical Field Demonstration of Mobile Weather Radar Set AN/TPS-41 at Fort Rucker, Alabama," ECOM-5572, August 1975.
9. Miers, B., G. Blackman, D. Langer, and N. Lorimier, "Analysis of SMS/GOES Film Data," ECOM-5573, September 1975.
10. Manquero, Carlos, Louis Duncan, and Rufus Bruce, "An Indication from Satellite Measurements of Atmospheric CO<sub>2</sub> Variability," ECOM-5574, September 1975.
11. Petracca, Carmine and James D. Lindberg, "Installation and Operation of an Atmospheric Particulate Collector," ECOM-5575, September 1975.
12. Avara, Elton P. and George Alexander, "Empirical Investigation of Three Iterative Methods for Inverting the Radiative Transfer Equation," ECOM-5576, October 1975.
13. Alexander, George D., "A Digital Data Acquisition Interface for the SMS Direct Readout Ground Station—Concept and Preliminary Design," ECOM-5577, October 1975.
14. Cantor, Israel, "Enhancement of Point Source Thermal Radiation Under Clouds in a Nonattenuating Medium," ECOM-5578, October 1975.
15. Norton, Colburn and Glenn Hoidale, "The Diurnal Variation of Mixing Height by Month over White Sands Missile Range, NM," ECOM-5579, November 1975.
16. Avara, Elton P., "On the Spectrum Analysis of Binary Data," ECOM-5580, November 1975.
17. Taylor, Fredrick J., Thomas H. Pries, and Chao-Huan Huang, "Optimal Wind Velocity Estimation," ECOM-5581, December 1975.
18. Avara, Elton P., "Some Effects of Autocorrelated and Cross-Correlated Noise on the Analysis of Variance," ECOM-5582, December 1975.
19. Gillespie, Patti S., R.L. Armstrong, and Kenneth O. White, "The Spectral Characteristics and Atmospheric CO<sub>2</sub> Absorption of the Ho<sup>+3</sup>:YLF Laser at 2.05 $\mu$ m," ECOM-5583, December 1975.
20. Novian, David J., "An Empirical Method of Forecasting Thunderstorms for the White Sands Missile Range," ECOM-5584, February 1976.
21. Avara, Elton P., "Randomization Effects in Hypothesis Testing with Autocorrelated Noise," ECOM-5585, February 1976.
22. Watkins, Wendell R., "Improvements in Long Path Absorption Cell Measurement," ECOM-5586, March 1976.

23. Thomas, Joe, George D. Alexander, and Marvin Dubbin, "SATTEL — An Army Dedicated Meteorological Telemetry System," ECOM-5587, March 1976.
24. Kennedy, Bruce W. and Delbert Bynum, "Army User Test Program for the RDT&E-XM-75 Meteorological Rocket," ECOM-5588, April 1976.
25. Barnett, Kenneth M., "A Description of the Artillery Meteorological Comparisons at White Sands Missile Range, October 1974 — December 1974 ('PASS' — Prototype Artillery [Meteorological] Subsystem)," ECOM-5589, April 1976.
26. Miller, Walter B., "Preliminary Analysis of Fall-of-Shot From Project 'PASS,'" ECOM-5590, April 1976.
27. Avara, Elton P., "Error Analysis of Minimum Information and Smith's Direct Methods for Inverting the Radiative Transfer Equation," ECOM-5591, April 1976.
28. Yee, Young P., James D. Horn, and George Alexander, "Synoptic Thermal Wind Calculations from Radiosonde Observations Over the Southwestern United States," ECOM-5592, May 1976.
29. Duncan, Louis D. and Mary Ann Seagraves, "Applications of Empirical Corrections to NOAA-4 VTPR Observations," ECOM-5593, May 1976.
30. Miers, Bruce T. and Steve Weaver, "Applications of Meterological Satellite Data to Weather Sensitive Army Operations," ECOM-5594, May 1976.
31. Sharenow, Moses, "Redesign and Improvement of Balloon ML-566," ECOM-5595, June 1976.
32. Hansen, Frank V., "The Depth of the Surface Boundary Layer," ECOM-5596, June 1976.